

JACARANDA MATHS QUEST
GENERAL MATHEMATICS 11

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JACARANDA MATHS QUEST
GENERAL
MATHEMATICS **11**
VCE UNITS 1 AND 2 | SECOND EDITION

JACARANDA MATHS QUEST
GENERAL
MATHEMATICS **11**
VCE UNITS 1 AND 2 | SECOND EDITION

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online 400

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ABOUT THIS RESOURCE

Jacaranda Maths Quest 11 General Mathematics VCE Units 1 & 2 Second Edition has been updated to enhance our in-depth coverage of the Study Design, with hundreds of new questions to expand students' understanding and improve learning outcomes. The Jacaranda Maths Quest series provides easy-to-follow text and is supported by a bank of resources for both teachers and students. At Jacaranda we believe that every student should experience success and build confidence, while those who want to be challenged are supported as they progress to more difficult concepts and questions.

Preparing students for exam success

Topic openers place mathematics in real-world contexts to drive engagement.

FREE access to studyON — our exam, study, revision and practice tool — is included with every title. studyON allows you to revise at the concept, topic, area of study or unit level.

Each subtopic concludes with carefully graded technology free and technology active questions.

The Manual for the TI-Nspire CAS calculator and Manual for the Casio ClassPad II calculator in the Prelims of your eBookPLUS provide step-by-step instructions on how to use CAS technology.

Features of the Maths Quest series

Questions and topics are sequenced from lower to higher levels of complexity; ideas and concepts are logically developed and questions are carefully graded, allowing every student to achieve success.

An extensive glossary of mathematical terms is provided in print and as a hover-over feature in the eBookPLUS.

TOPIC 3
Financial arithmetic

3.1 Overview
3.1 Introduction
Bank interest today is very different to what it originally was thousands of years ago. The basic principle, however, remains the same. The early loans and interest, around 3000 to 2000 BC, were used in agriculture. Loans were made in wool, grain, animals and such in farmers. Since one would need to generate a plant with over 100 new seeds after the harvest, the farmer farmers to pay back their loans with interest. When animals were loaned, interest was paid by slaughtering new-born animals.
Aristotle noted that money was sterile and that, unlike grain or animals, it did not produce more money, an interest was not earned in the same way. The church scholar known as the Scholastic school Aristotle's daughter in 1100 to 1500 AD. They made the first attempt at the science of economics and their main concern was to incorporate interest.

Resources
Interactivity: Stocks and currency (in text)

studyON
Interactivity: Where prices and dividends (concept summary and practice questions)

Exercise 3.3 Financial applications of ratios and percentages
Understand the directed, give all answers to the following questions correct to 2 decimal places or the nearest other appropriate.
1. Calculate the dividend payable per share for a company with:
a. 220 000 shares, when \$190 000 of its annual profit is distributed to the shareholders
b. 4420 211 shares, when \$596 000 of its annual profit is distributed to the shareholders
c. 20 400 shares, when \$20 200 of its annual profit is distributed to the shareholders.
2. How many shares are in a company that declares a dividend of:
a. 2.0¢ per share when \$160 000 of its annual profit is distributed
b. \$2.24 per share when \$205 540 of its annual profit is distributed
c. \$2.24 per share when \$9 853 000 of its annual profit is distributed
d. \$14.0¢ per share when \$152 950 of its annual profit is distributed
3. **NOTE** Calculate the necessary dividends of the following

Manuals
MANUAL FOR THE TI-Nspire CAS CALCULATOR
MANUAL FOR THE CASIO ClassPad II CALCULATOR

3.1.2 Kick off with CAS
Calculating interest with CAS
CAS can be used to quickly and easily evaluate formulas when given specific values. The formula to calculate simple interest is $I = \frac{Prt}{100}$ where I is the interest earned, P is the principal, r is the rate of interest and T is the time.
1. Using CAS, define and save the formula for simple interest.
2. Use the formula to calculate the missing values in the following situations.
a. $P = \$5000$, $r = 4\%$, $T = 2$ years
b. $I = \$945$, $r = 4.5\%$, $T = 3$ years
c. $I = \$748$, $P = \$5500$, $T = 4$ years
d. $I = \$113.50$, $P = \$330$, $r = 3.8\%$
The formula to calculate compound interest is $A = P(1 + \frac{r}{n})^{nt}$, where A is the final amount, P is the principal, r is the rate of interest and n is the number of interest-bearing periods.
3. Using CAS, define and save the formula for compound interest.
4. Use the formula to calculate the missing values in the following situations.
a. $P = \$5000$, $r = 3\%$, $n = 2$ years
b. $A = \$6000$, $r = 5\%$, $n = 4$ years
c. $A = \$2812.16$, $P = \$2500$, $n = 3$ years
d. $A = \$3506.07$, $P = \$3500$, $r = 5\%$
The value of the final amount for simple interest can be calculated by summing I and P .
5. Use CAS to help you complete the following table comparing simple and compound interest.

Principal	Rate of interest	Time period	Simple interest final amount	Compound interest final amount
\$4000	4%	3 years		
\$2500	3.5%		\$2850	
\$3000		2 years		\$3533.52

3.7 Review: exam practice
A summary of this topic is available in the Resources section of your eBookPLUS at www.jacplus.com.au.
Multiple choice
1. The price of petrol increased from 118.4 cents to 130.9 cents. The percentage change is:
a. 10.6% b. 9.5% c. 9.0% d. 1.1% e. 12.5%
2. A basketball ring is sold for \$28.50. If this represents a 24% reduction from the RRP, the original price was:
a. \$36.25 b. \$18.75 c. \$35.50 d. \$37.50 e. \$26.67
3. A company has 641 183 shares. When \$958 500 of its annual profit is distributed to the shareholders, the dividend payable per share is:
a. \$6.87 b. \$1.49 c. \$6.73 d. \$6.71 e. \$0.49
4. How many shares are in a company that declares a dividend of 32 cents per share when \$450 000 of its annual profit is distributed?
a. 144 000 b. 14 063 c. 6617 d. 30 600 000 e. 1406 200
5. The price-to-earnings ratio for a company with a share price of \$2.40 and a dividend of 87 cents is:
a. 2.90 b. 2.76 c. 6.03 d. 3.27 e. 6.3625
6. When the simple interest formula is rearranged to find r , the correct formula is:
a. $r = \frac{100P}{PT}$ b. $r = \frac{P}{PT}$ c. $r = \frac{100}{PT}$ d. $r = \frac{100P}{PT}$
7. Which of the following companies has the lowest share price?
a. Company A with a price-to-earnings ratio of 10.4 and a dividend of \$1.87
b. Company B with a price-to-earnings ratio of 21.3 and a dividend of 70 cents
c. Company C with a price-to-earnings ratio of 14.8 and a dividend of 79 cents
d. Company D with a price-to-earnings ratio of 15.75 and a dividend of 97 cents
e. Company E with a price-to-earnings ratio of 17.7 and a dividend of \$1.33
8. A business offers a 6.5% discount for customers who pay in cash. How much would a customer pay if they paid their bill of \$24 in cash?
a. \$16.59 b. \$218.48 c. \$227.41 d. \$261.50 e. \$277.20
9. A new racing bike priced at \$6000 is sold for an 8% profit. The cost of \$5900 plus 13 monthly instalments of \$280. The effective rate of interest is:
a. 14.36% b. 40.91% c. 60.96% d. 204.61% e. 2%
10. Meredith walks dogs at the weekend. She charges \$14.00 per dog plus \$6.00 per hour. She offers her clients a 5% discount for paying in cash. How much would she charge for someone paying cash to walk 3 dogs for 2 hours?
a. \$51.30 b. \$11.10 c. \$54 d. \$2.70 e. \$48

Some simple expansions include:
 $(a + b)(c + d) = ac + ad + bc + bd$
 $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$
 $(a - b)^2 = a^2 - 2ab + b^2$
 $(a + b)(a - b) = a^2 - b^2$

WORKED EXAMPLE 1
Expand $2(4x - 3)^2 - (x - 2)(x + 2) + (x + 5)(2x - 1)$ and state the coefficient of the term in x .
THINK
1. Expand each pair of brackets.
2. Note: The first term contains a perfect square, the second a difference of two squares and the third a quadratic trinomial.
3. Expand fully, taking care with signs.
4. Collect like terms together.
5. State the answer.
NOTE: Read the question again to ensure the answer given is as requested.
 $2(4x - 3)^2 - (x - 2)(x + 2) + (x + 5)(2x - 1)$
 $= 2(16x^2 - 24x + 9) - (x^2 - 4) + (2x^2 + 9x - 5)$
 $= 32x^2 - 48x + 18 - x^2 + 4 + 2x^2 + 9x - 5$
 $= 33x^2 - 39x + 17$
The expansion gives $33x^2 - 39x + 17$ and the coefficient of x is -39 .

THINK
1. On a Calculator page, press $2 \times (4x - 3)^2 - (x - 2)(x + 2) + (x + 5)(2x - 1)$ and enter.
2. The answer appears on the screen: $33x^2 - 39x + 17$.

WRITE
1. On the Main screen, complete the expansion: $2(4x - 3)^2 - (x - 2)(x + 2) + (x + 5)(2x - 1)$. Then press ENTER.
2. The expansion gives $33x^2 - 39x + 17$.

CASIO/THINK
1. On the Main screen, complete the expansion: $2(4x - 3)^2 - (x - 2)(x + 2) + (x + 5)(2x - 1)$. Then press EXE.
2. The expansion gives $33x^2 - 39x + 17$.

TOPIC 3 — Financial arithmetic
Exercise 3.2 — Percentage change
1. $(1.25 - 1) \times 100 = 0.25 \times 100 = 25\%$ increase
2. $(1.17 - 1) \times 100 = 0.17 \times 100 = 17\%$ increase
3. $(1.22 - 1) \times 100 = 0.22 \times 100 = 22\%$ increase
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Each topic opens with an engaging Kick off with CAS activity designed to stimulate students' interest and curiosity and to highlight the important applications of CAS technology in developing deep understanding of the mathematical concepts presented.

At the conclusion of each topic, Review questions allow you to revise the entire topic. Short-answer technology free questions allow you to consolidate your learning. Technology active questions are provided through multiple-choice and extended-response questions.

Fully worked examples in the Think/Write format provide guidance and are linked to questions.

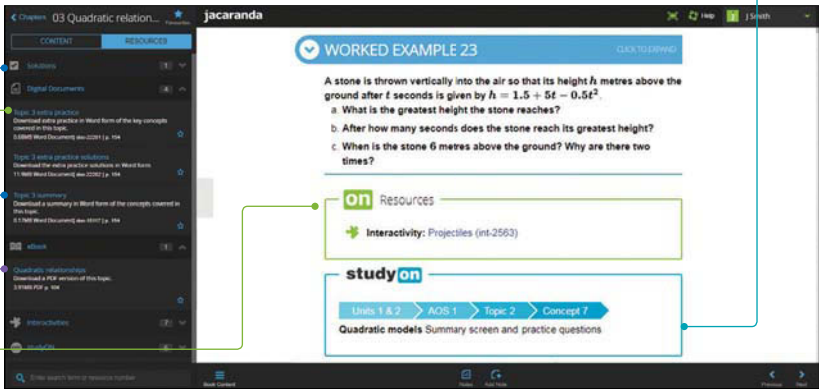
Selected worked examples demonstrate the use of CAS calculators.

FREE fully worked solutions are provided, enabling students to get help where they need it, whether at home or in the classroom — help at the point of learning is critical. Answers are provided at the end of each topic in the print and offline PDF.

eBookPLUS features

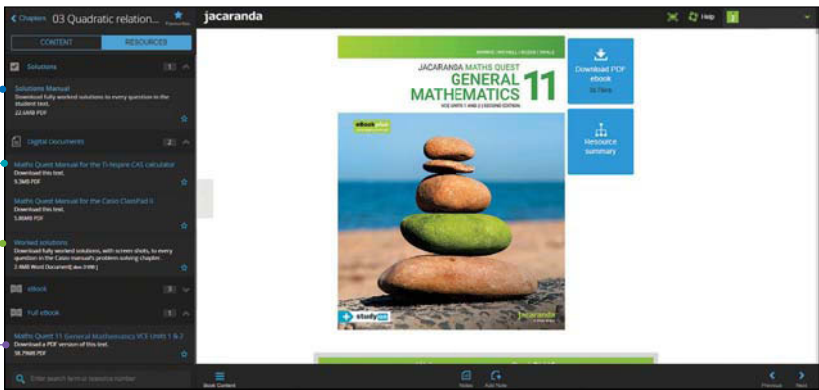
- Fully worked solutions for every question
- Digital documents: downloadable extra practice questions and fully worked solutions covering all key concepts in the topic
- Topic summaries in downloadable format to assist in study and exam preparation
- A downloadable PDF of the entire topic of the print text
- Interactivities and video eLessons placed at the point of learning to enhance understanding and correct common misconceptions

Concept summary links to studyON for study, revision and exam practice



In the Prelims section of your eBookPLUS

- A downloadable PDF of the entire solutions manual, containing fully worked solutions for every question in the text
- FREE copies of the *Maths Quest Manual for the TI-Nspire CAS calculator* and the *Maths Quest Manual for the Casio ClassPad II calculator*
- Downloadable fully worked solutions, with screenshots, to every question in the Casio manual's problem-solving chapter.



A downloadable PDF of the entire print text

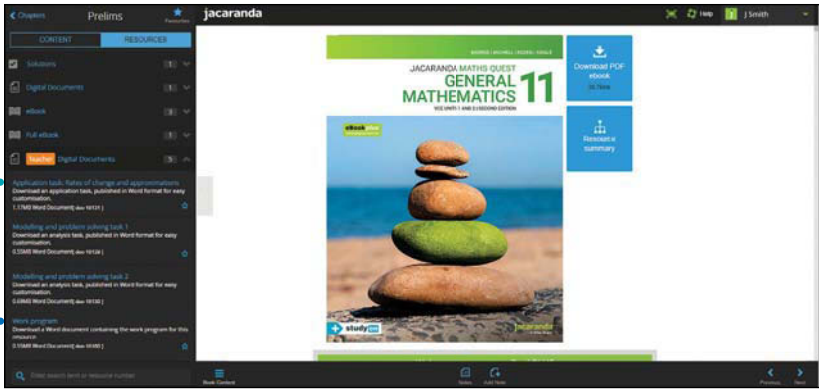
Additional resources for teachers available in the eGuidePLUS

- In the Resources tab of every topic there are two topic tests in downloadable, customisable Word format with worked solutions.



In the Prelims section of the eGuidePLUS

- Additional resources for teachers include an application task and two problem solving and modelling tasks.
- A work program is provided to assist with classroom planning.



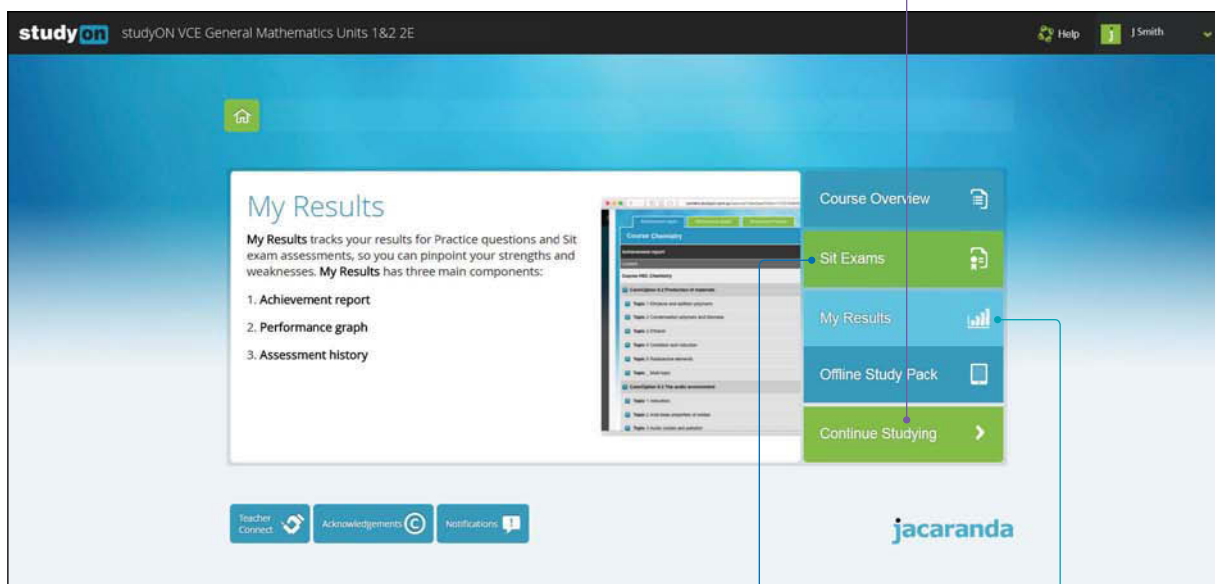
studyON – an invaluable exam preparation tool

studyON provides a complete study solution. An interactive and highly visual online study, revision and exam practice tool, it is designed to help students and teachers maximise exam results.

Concept summary screens and interactivities summarise key concepts and help prevent misconceptions.

Direct links from the eBookPLUS help scaffold students' understanding and study practices.

The studyON question hierarchy allows students in the *Continue Studying* feature to revise across the entire course, or to drill down to concept level for a more granular set of questions.



- The Sit Exams feature allows students to sit past VCAA exams (Units 3 & 4) or topic tests (Units 1 & 2) in timed, exam-like situations.
- Students have access to 1000+ past VCAA questions or custom-authored practice questions at a concept, topic or entire course level, and receive immediate feedback for every question.

studyON's built-in progress tracker enables self-diagnosis of strengths and weaknesses at a topic and concept level, so students know exactly what needs extra revision and can sit their exams with confidence.

studyON Teacher edition is a powerful diagnostic tool



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Allows teachers to monitor students' activities and results to pinpoint strengths and weaknesses. Armed with evidence-based insights, teachers can intervene at the right time.

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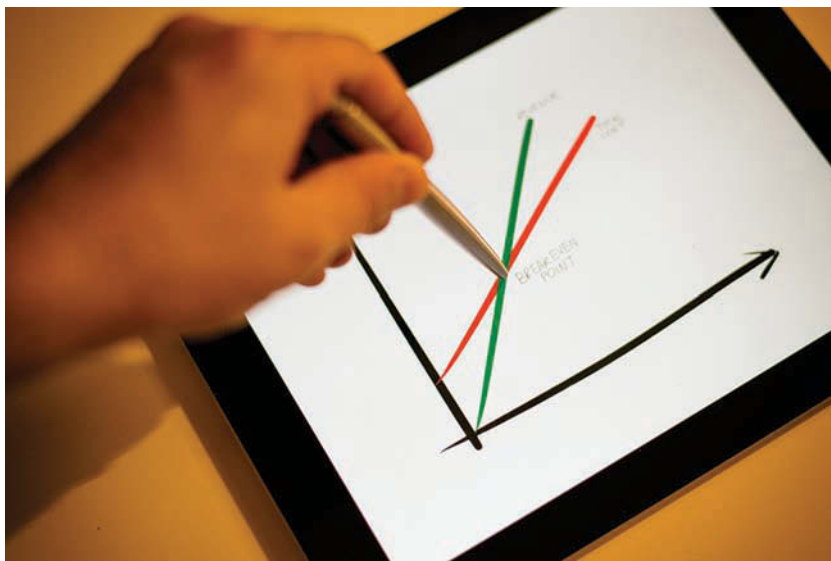
TOPIC 1

Linear relations and equations

1.1 Overview

1.1.1 Introduction

Linear equations have been around for over 4000 years. A simple 2×2 linear equation system with two unknowns was first solved by the Babylonians. Around 200 BC, the Chinese demonstrated the ability to solve a 3×3 system of equations. However, it wasn't until the 17th century that progress was made in linear algebra by the inventor of calculus, Leibniz. This was followed by work by Cramer, and later Gauss. Linear equations themselves were invented in 1843 by Irish mathematician



Sir William Rowan Hamilton. He made important contributions to mathematics and his work was also used in quantum mechanics. Sir William Hamilton was reputedly a genius: at the young age of 13, he reportedly spoke 13 languages, and at 22 he was a professor at the University of Dublin. His work has been applied in many fields, given that there are many situations in which there is a direct relationship between two variables. Classic examples are of water being added to a tank at a constant rate or a taxi trip being charged at a constant rate per kilometre. A linear equation to model the cost of a taxi trip can be used to compare one taxi company to another. The break-even point refers to the point at which the cost is the same for each taxi company. This is the point at which the two linear graph intersect and it can be found using a graphical technique, or using substitution or elimination techniques of simultaneous equations.

LEARNING SEQUENCE

- 1.1 Overview
- 1.2 Linear relations
- 1.3 Solving linear equations
- 1.4 Developing linear equations
- 1.5 Simultaneous linear equations
- 1.6 Problem solving with simultaneous equations
- 1.7 Review: exam practice

Fully worked solutions for this topic are available in the Resources section of your eBookPLUS at www.jacplus.com.au

1.1.2 Kick off with CAS

Linear equations with CAS

Linear equations link two variables in a linear way such that as one variable increases or decreases, the other variable increases or decreases at a constant rate. We can use CAS to quickly and easily solve linear equations.

1. Use CAS to solve the following linear equations.

- $6x = 24$
- $0.2y - 3 = 7$
- $5 - 3p = -17$

CAS can also be used to solve linear equations involving fractions and brackets.

2. Use CAS to solve the following linear equations.

- $\frac{5x + 3}{2} = 14$
- $0.5(y + 6) = 9$
- $\frac{3(t + 1)}{2} = 12$

A literal equation is an equation containing several pronumerals or variables. We can solve literal equations by expressing the answer in terms of the variable we are looking to solve for.

3. Use CAS to solve the following literal equations for a .

- $ax + b = 2m$
- $m(a - 3b) = 2t$
- $\frac{3am - 4}{t} = cd$

4. a. The equation $A = \frac{1}{2}bh$ is used to find the area of a triangle given the base length and the height. Use CAS to solve $A = \frac{1}{2}bh$ for b .

- Use your answer to part a to find the base lengths of triangles with the following heights and areas.
 - Height = 5 cm, area = 20 cm²
 - Height = 6.5 cm, area = 58.5 cm²



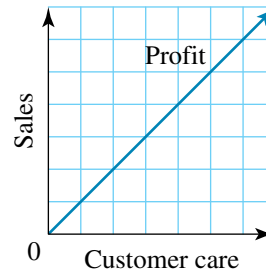
on Resources

Please refer to the Resources section in the Prelims of your eBookPLUS for a comprehensive step-by-step guide on how to use your CAS technology.

1.2 Linear relations

1.2.1 Identifying linear relations

A **linear relation** is a relationship between two **variables** that when plotted give a straight line. Many real-life situations can be described by linear relations, such as water being added to a tank at a constant rate, or money being saved when the same amount of money is deposited into a bank at regular time intervals.



When a linear relation is expressed as an equation, the highest power of both variables in the equation is 1.

WORKED EXAMPLE 1

Identify which of the following equations are linear.

a. $y = 4x + 1$

b. $b = c^2 - 5c + 6$

c. $y = \sqrt{x}$

d. $m^2 = 6(n - 10)$

e. $d = \frac{3t + 8}{7}$

f. $y = 5^x$

THINK

- a. 1. Identify the variables.
2. Write the power of each variable.
3. Check if the equation is linear.

- b. 1. Identify the two variables.
2. Write the power of each variable.
3. Check if the equation is linear.

- c. 1. Identify the two variables.
2. Write the power of each variable.
Note: A square root is a power of $\frac{1}{2}$.
3. Check if the equation is linear.

- d. 1. Identify the two variables.
2. Write the power of each variable.
3. Check if the equation is linear.

WRITE

- a. y and x
 y has a power of 1.
 x has a power of 1.
Since both variables have a power of 1, this is a linear equation.
- b. b and c
 b has a power of 1.
 c has a power of 2.
 c has a power of 2, so this is not a linear equation.
- c. y and x
 y has a power of 1.
 x has a power of $\frac{1}{2}$.
 x has a power of $\frac{1}{2}$, so this is not a linear equation.
- d. m and n
 m has a power of 2.
 n has a power of 1.
 m has a power of 2, so this is not a linear equation.

- | | |
|---|---|
| <p>e. 1. Identify the two variables.</p> <p>2. Write the power of each variable.</p> <p>3. Check if the equation is linear.</p> | <p>e. d and t</p> <p>d has a power of 1.</p> <p>t has a power of 1.</p> <p>Since both variables have a power of 1, this is a linear equation.</p> |
| <p>f. 1. Identify the two variables.</p> <p>2. Write the power of each variable.</p> <p>3. Check if the equation is linear.</p> | <p>f. y and x</p> <p>y has a power of 1.</p> <p>x is the power.</p> <p>Since x is the power, this is not a linear equation.</p> |

1.2.2 Rules for linear relations

Rules define or describe relationships between two or more variables. Rules for linear relations can be found by determining the **common difference** between consecutive terms of the pattern formed by the rule.

Consider the number pattern 4, 7, 10 and 13. This pattern is formed by adding 3 (the common difference is 3). If each number in the pattern is assigned a term number as shown in the table, then the expression to represent the common difference is $3n$ (i.e. $3 \times n$).

Term number, n	1	2	3	4
$3n$	3	6	9	12

Each term in the number pattern is 1 greater than $3n$, so the rule for this number pattern is $3n + 1$.

If a rule has an equals sign, it is described as an equation. For example, $3n + 1$ is referred to as an expression, but if we define the term number as t , then $t = 3n + 1$ is an equation.

WORKED EXAMPLE 2

Find the equations for the linear relations formed by the following number patterns.

- a. 3, 7, 11, 15 b. 8, 5, 2, -1

THINK

- a. 1. Determine the common difference.
2. Write the common difference as an expression using the term number n .
3. Substitute any term number into $4n$ and evaluate.
4. Check the actual term number against the one found.
5. Add or subtract a number that would result in the actual term number.
6. Write the equation for the linear relation.
- b. 1. Determine the common difference.
2. Write the common difference as an expression using the term number n .

WRITE

- a. $7 - 3 = 4$
- $15 - 11 = 4$
- $4n$
- $n = 3$
- $4 \times 3 = 12$
- The actual 3rd term is 11.
- $12 - 1 = 11$
- $t = 4n - 1$
- b. $5 - 8 = -3$
- $2 - 5 = -3$
- $-3n$

- Substitute any term number into $-3n$ and evaluate.
- Check the actual term number against the one found.
- Add or subtract a number that would result in the actual term number.
- Write the equation for the linear relation.

$$n = 2$$

$$-3 \times 2 = -6$$

The actual 2nd term is 5.

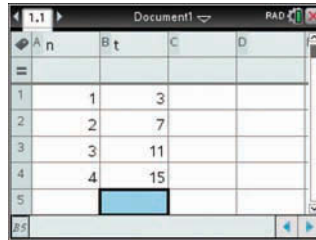
$$-6 + 11 = 5$$

$$t = -3n + 11$$

TI | THINK

- On a Lists & Spreadsheet page, label the first column n for the term number, and the second column t for the term value. As there are four terms given in this sequence, enter the numbers 1 to 4 in the first column. Enter the terms of the given sequence in the second column.
- On a Calculator page, press MENU and select:
 - Statistics
 - 1: Stat Calculations
 - 3: Linear Regression ($mx + b$) ...
 Select n as the X List and t as the Y List, then select OK.

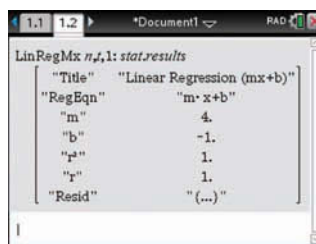
WRITE



n	t
1	3
2	7
3	11
4	15



- Interpret the output on the screen.



Parameter	Value
Title	Linear Regression (mx+b)
RegEqn	m · x + b
m	4.
b	-1.
r	1.
r ²	1.
Resid	(...)

The equation is given in the form $y = mx + b$, where $y = t_n$, $m = 4$, $x = n$ and $b = -1$.

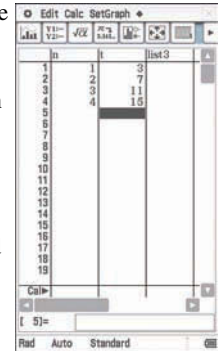
- State the answer.

The equation is $t_n = 4n - 1$.

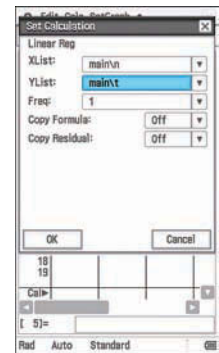
CASIO | THINK

- On a Statistics screen, change the label of "list 1" to n and that of "list 2" to t . As there are four terms given in this sequence, enter the numbers 1 to 4 in the first column. Enter the terms of the given sequence in the second column.
- On the Statistics screen, select:
 - Calc
 - Regression
 - Linear Reg
 Select main\ n as the XList and main\ t as the YList, then select OK.

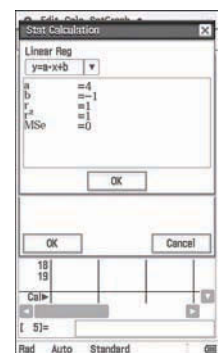
WRITE



list 1	list 2
1	3
2	7
3	11
4	15



- Interpret the output on the screen.



Parameter	Value
Linear Reg	y = a · x + b
a	= 4
b	= -1
r	= 1
r ²	= 1
MISr	= 0

The equation is given in the form $y = ax + b$, where $y = t_n$, $a = 4$, $x = n$, and $b = -1$.

- State the answer.

The equation is $t_n = 4n - 1$.

Note: It is good practice to substitute a second term number into your equation to check that your answer is correct.

1.2.3 Transposing linear equations

If we are given a **linear equation** between two variables, we are able to **transpose** this relationship. That is, we can change the equation so that the variable on the right-hand side of the equation becomes the stand-alone variable on the left-hand side of the equation.

WORKED EXAMPLE 3

Transpose the linear equation $y = 4x + 7$ to make x the subject of the equation.

THINK

1. Isolate the variable on the right-hand side of the equation (by subtracting 7 from both sides).
2. Divide both sides of the equation by the coefficient of the variable on the right-hand side (in this case 4).
3. Transpose the relation by interchanging the left-hand side and the right-hand side.

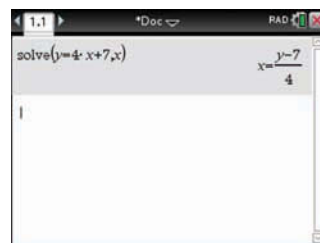
WRITE

$$\begin{aligned} y - 7 &= 4x + 7 - 7 \\ y - 7 &= 4x \\ \frac{y - 7}{4} &= \frac{4x}{4} \\ \frac{y - 7}{4} &= x \\ x &= \frac{y - 7}{4} \end{aligned}$$

TI | THINK

1. On a Calculator page, press MENU and select:
3: Algebra
1: Solve
Complete the entry line as solve ($y = 4 \cdot x + 7, x$) then press ENTER.

WRITE



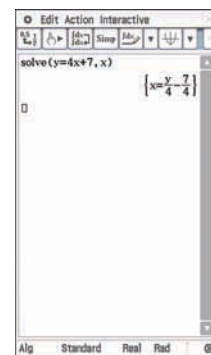
2. The answer appears on the screen.

$$x = \frac{y - 7}{4}$$

CASIO | THINK

1. On the Main screen, complete the entry line as:
solve ($y = 4x + 7, x$)
then press EXE.

WRITE



2. The answer appears on the screen.

$$x = \frac{y - 7}{4}$$

on Resources

Interactivity: Transposing linear equations (int-6449)

study on

Units 1 & 2 > AOS 1 > Topic 1 > Concept 3 > **Transposition** Concept summary and practice questions

Exercise 1.2 Linear relations

1. **WE1** Identify which of the following equations are linear.
 - a. $y^2 = 7x + 1$
 - b. $t = 7x^3 - 6x$
 - c. $y = 3(x + 2)$
 - d. $m = 2^{x+1}$
 - e. $4x + 5y - 9 = 0$

2. Bethany was asked to identify which equations from a list were linear. The following table shows her responses.

Equation	Bethany's response
$y = 4x + 1$	Yes
$y^2 = 5x - 2$	Yes
$y + 6x = 7$	Yes
$y = x^2 - 5x$	No
$t = 6d^2 - 9$	No
$m^3 = n + 8$	Yes



- a. Insert another column into the table and add your responses identifying which of the equations are linear.
- b. Provide advice to Bethany to help her to correctly identify linear equations.
3. Identify which of the following are linear equations.
- a. $y = 2t + 5$ b. $x^2 = 2y + 5$ c. $m = 3(n + 5)$ d. $d = 80t + 25$
- e. $y^2 = 7x + 12$ f. $\sqrt{y} = x + 5$ g. $s = \frac{100}{t}$
4. Samson was asked to identify which of the following were linear equations. His responses are shown in the table.

Equation	Samson's response
$y = 5x + 6$	Yes, linear
$y^2 = 6x - 1$	Yes, linear
$y = x^2 + 4$	Not linear
$y^3 = 7(x + 3)$	Yes, linear
$y = \frac{1}{2}x + 6$	Yes, linear
$\sqrt{y} = 4x + 2$	Yes, linear
$y^2 + 5x^3 + 9 = 0$	Not linear
$10y - 11x = 12$	Yes, linear

- a. Based on Samson's responses, would he state that $6y^2 + 7x = 9$ is linear? Justify your answer.
- b. What advice would you give to Samson to ensure that he can correctly identify linear equations?
5. **WE2** Find the equations for the linear relations formed by the following number patterns.
- a. 2, 6, 10, 14, 18, ... b. 4, 4.5, 5, 5.5, 6, ...
6. Jars of vegetables are stacked in ten rows. There are 8 jars in the third row and 5 jars in the sixth row. The number of jars in any row can be represented by a linear relation.
- a. Find the common difference.
- b. Find an equation that will express the number of jars in any of the ten rows.
- c. Determine the total number of jars of vegetables.

7. A number pattern is formed by multiplying the previous term by 1.5. The first term is 2.
- Find the next four terms in the number pattern.
 - Could this number pattern be represented by a linear equation? Justify your answer.
8. Find equations for the linear relations formed by the following number patterns.
- 3, 7, 11, 15, 19, ...
 - 7, 10, 13, 16, 19, ...
 - 12, 9, 6, 3, 0, -3, ...
 - 13, 7, 1, -5, -11, ...
 - 12, -14, -16, -18, -20, ...
9. Consider the following number pattern: 1.2, 2.0, 2.8, 3.6, 4.4, ...
- Find the first common difference.
 - Could this number pattern be represented by a linear equation? Justify your answer.
10. **WE3** Transpose the linear equation $y = 6x - 3$ to make x the subject of the equation.
11. Transpose the linear equation $6y = 3x + 1$ to make x the subject of the equation.
12. Transpose the following linear equations to make x the subject.
- $y = 2x + 5$
 - $3y = 6x + 8$
 - $p = 5x - 6$
13. Water is leaking from a water tank at a linear rate. The amount of water, in litres, is measured at the start of each day. At the end of the first day there are 950 litres in the tank, and at the end of the third day there are 850 litres in the tank.
- Complete the following table.

Day	1	2	3	4	5
Amount of water (L)	950		850		

- Determine the amount of water that was initially in the tank (i.e. at day 0).
- Determine an equation that finds the amount of water, w , in litres, at the end of any day, d .



14. At the start of the year Yolanda has \$1 500 in her bank account. At the end of each month she deposits an additional \$250.
- How much, in dollars, does Yolanda have in her bank account at the end of March?
 - Find an equation that determines the amount of money, A , Yolanda has in her bank account at the end of each month, m .
 - At the start of the following year, Yolanda deposits an additional \$100 each month. How does this change the equation found in part **b**?



15. On the first day of Sal's hiking trip, she walks halfway into a forest. On each day after the first, she walks exactly half the distance she walked the previous day. Could the distance travelled by Sal each day be described by a linear equation? Justify your answer.
16. Catalina is a runner whose goal is to run a total of 350 km over 5 weeks to raise money for charity.
- If each week she runs 10 km more than she did on the previous week, how far does she run in week 3?
 - Find an equation that determines the distance Catalina runs each week.
17. Using CAS or otherwise, determine an equation that describes the number pattern shown in the table below.

Term number	1	2	3	4	5
Value	-4	-2	0	2	4

18. The terms in a number sequence are found by multiplying the term number, n , by 4 and then subtracting 1. The first term of the sequence is 3.
- Find an equation that determines the terms in the sequence.
 - Using CAS or otherwise, find the first 10 terms of the sequence.
 - Show that the common difference is 4.



1.3 Solving linear equations

1.3.1 Solving linear equations with one variable

To solve linear equations with one variable, all operations performed on the variable need to be identified in order, and then the opposite operations need to be performed in reverse order.

In practical problems, solving linear equations can answer everyday questions such as the time required to have a certain amount in the bank, the time taken to travel a certain distance, or the number of participants needed to raise a certain amount of money for charity.

WORKED EXAMPLE 4

Solve the following linear equations to find the unknowns.

a. $5x = 12$

b. $8t + 11 = 20$

c. $12 = 4(n - 3)$

d. $\frac{4x - 2}{3} = 5$

THINK

a. 1. Identify the operations performed on the unknown.

2. Write the opposite operation.

3. Perform the opposite operation on both sides of the equation.

4. Write the answer in its simplest form.

WRITE

a. $5x = 5 \times x$

So the operation is $\times 5$.

The opposite operation is $\div 5$.

Step 1 ($\div 5$):

$$5x = 12$$

$$\frac{5x}{5} = \frac{12}{5}$$

$$x = \frac{12}{5}$$

$$x = \frac{12}{5}$$